REVIEW ARTICLE OPEN ACCESS

Performance Of A Combined Shewhart-Cusum Control Chart With Binomial Data For Large Shifts In The Process Mean

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ABSTRACT
The purpose of this study is to investigate the performance of a combined Shewhart-CUSUM chart for binomial data, when compared to individual Shewhart and CUSUM control charts for large shifts in the process mean. The comparison is performed by analyzing several ARL (Average Run Length) measures. The ARL of the combined control charts is identified through Monte Carlo simulations. The results confirm the effectiveness of the combined charts to increase the sensitivity of CUSUM schemes. The Shewhart limits make it possible to indicate large changes in the process mean. And, there is a region of process shifts where the combined chart displays better performance than in individual procedures. Therefore, for the detection of only large alterations in the process mean, an individual CUSUM or a Shewhart chart is recommended.
***Keywords* - Combined Shewhart-CUSUM chart; Binomial data; Average Run Length.**

I. INTRODUCTION

The traditional Shewart charts are the most widely known and applied. The simplicity of the decision rule conditioned merely on analyzing the last point signaled on the chart facilitates evaluating if such a point signals special cause. However, the major disadvantage is utilizing only the information about the process contained in the last point and ignoring any information given by the entire sequence of points. This characteristic makes these control charts insensitive to slight changes in the process, on the order of 1.5 standard deviations or less [1].

Although very efficient, these charts are not the only tools available. In some cases, other types of control charts may advantageously complement or substitute the traditional Shewhart scheme. Such is the case of the CUSUM (Cumulative Sum) and the EWMA (Exponentially Weighted Moving Average) control charts. These charts are indicated to monitor processes subject to small shifts, whose decision concerning the statistical control state is based on information accumulated from previous samples and not only the last one. With these charts, it is possible to signal small shifts more quickly, as well as identify the approximate moment in time in which a change in the process occurs. However, if the magnitude of the change were unknown or if there were an alteration over time, none of the previously-mentioned charts standing individually will offer adequate performance concerning both small and large shifts. But, it is possible to combine multiple charts in order to monitor diverse magnitudes of change, adding Shewhart control limits to the CUSUM control chart in order to detect both small and large shifts [2].

The purpose of this study is to investigate the performance of a combined Shewhart-CUSUM chart for data with a binomial distribution when compared to individual Shewhart and CUSUM control charts for large shifts in the process mean.

This paper is structured as follows. Section 2 briefly describes the combined Shewhart-CUSUM chart. Section 3 provides the methodological procedures used. Section 4 presents the results and analysis. Finally, in Section 5 there are concluding remarks.

II. COMBINED SHEWHART-CUSUM CHART

A combined Shewhart-CUSUM chart incorporates observed values, Shewhart control limits, the CUSUM statistic, and the CUSUM decision interval all on the same axis. More importantly, it is the probabilistic combination of two control charts into a single chart where control limits must be recalculated taking into account the sensitivity of false alarm rates to combination schemes. The combination of charts was proposed to improve the sensitivity of the CUSUM over a wide range of shifts in the process mean and has wide practical applications [1] [3] [4] [5] [6] [7] [8]. The literature has presented few studies relating to combined charts for attributes data. Numerical results relative to the unilateral combined Shewhart-CUSUM chart for attributes considering the Poisson distribution may be found in [9] [10] [11], as well as binomial distribution approaches from [12] [13].

A combined Shewhart-CUSUM chart generally assumes four control lines: the upper and lower Shewhart limits and the decision interval for the CUSUM. It should also show the values of the statistics corresponding to the samples (observed or standardized) and the values of the cumulative sums (positive and negative). Because of this, the analysis can become unclear, and some reductions can be made according to the case being treated. For example, if it is only desired to investigate a problem such as increasing the proportion of defective parts, an upper unilateral chart may be sufficient [14].

The unilateral combined Shewhart-CUSUM chart for binomial data incorporates a Shewhart *np* control chart for the number of nonconforming units and a binomial CUSUM chart on the same axis. The Shewhart *np* control chart is extensively utilized to monitor processes which produce a certain percentage of defective parts [15] [16] [17]. It is widely used in factories in which the use of the statistical process control is in initial implementation phase [17]. A binomial CUSUM chart examines the number of nonconformities accumulated in a sequence of samples. Its objective is to identify either increases or decreases in the number of nonconformities. Thus, a unilateral binomial CUSUM chart may be applied to detect an increase in the expected value of the rate of nonconforming items, from the nominal value p_0 to p_1; p_0 is the proportion of nonconforming items considered to be within statistical control, while p_1 is the pre-established proportion outside of statistical control to be detected [12] [18] [4].

Thus, if Y_i were a series of independent samples with $i \in Z^+$ of size n and with binomial distribution, the combined Shewhart-CUSUM binomial chart is obtained through plotting the statistics C_i and X_i, with respect to the sequence of samples Y_i,

$$C_i = \max(0, C_{i-1} + X_i - k), \quad (1)$$

where C_i is the CUSUM statistic with $C_0 = u, 0 \leq u \leq h$; X_i is the number of nonconforming items in the sample Y_i; k is the CUSUM reference value and depends upon the magnitude of the change one desires to detect, and h is the CUSUM control limit. The reference value k for the CUSUM chart is determined by the rate of acceptable counts (np_0) and the rate of counts that one wishes to detect (np_1). [19] proved that for a binomial CUSUM, the k value obtained through the equation below may be considered an optimal value (in terms of the ARL) in detecting an upward shift of magnitude in the parameter p [12].

$$k = \frac{n \ln((1-p_0)/(1-p_1))}{\ln((1-p_0)/(1-p_1)) - \ln(p_0/p_1)} \quad (2)$$

The Upper Control Limit (UCL) from the conventional Shewhart *np* chart is expressed as

$$UCL = np + 3\sqrt{np_0(1-p_0)} \ . \quad (3)$$

These limits are obtained through an approximation to the normal distribution for the value of the standard error. The use of the approximation is standard procedure in this area, however it is suggested that exact limits be calculated utilizing the binomial distribution.

The two lines C_i and X_i, and the two control limits, h and UCL (Shewhart Upper Control Limit), are then plotted. If C_i goes beyond h or X_i goes beyond the UCL, this signifies that the process is not in statistical control.

The Average Run Length (ARL) corresponds to the average value of the number of observations which must be plotted to indicate a condition out of statistical control. ARL_0 indicates the average number of samples collected until the emission of a signal during the period under control, often refered to as a false alarm. ARL_1 represents the average number of samples collected until the emission of a signal that indicates a situation that is truly out of control.

The upper limit h of the CUSUM is determined as a function of ARL_0 and ARL_1. The precise relationship among the three parameters (k, h and ARL) is not straightforward. There are various procedures used in the literature to calculate the ARL for a binomial CUSUM. The most common is as a Markov process [20] [21] [22]. Other procedures are based on Siegmund's proposal to Wald's approximations [23] [24], and the work of [19] [25].

Performance measurements for combined charts can be calculated in several ways already well documented in the literature. Markov chains were applied by [5] [7] [12]; to discrete variables with a Poisson distribution, and simulation procedures by [14] for continuous variables. In [1] [26] [27] and [4] all approached the combination of control charts while considering the probability of false alarms. This means that adjusting UCL and h to allocate the Type I error of the combination between the Shewhart chart and the CUSUM chart. If UCL is tightened, h must be relaxed. This will make the combined chart more sensitive to large mean shifts. Similarly, if UCL is loosened, h must be tightened, and the scheme will be more powerful for detecting small shifts [4].

In general, if there are 2 control charts monitoring the same variable and if each chart is characterized by the same probability of type I error, then the combined probability (type I error) for the combined control charts is $\alpha' = 1-(1-\alpha)^2$, where $(1-\alpha)^2$ is the probability that the two variables are simultaneously within their control limits. Thus, when two different charts monitor the same variable, the false alarm rate of the combined chart α_{cs} could be a combination of the individual rates of each, α_s and α_c for Shewhart (S) and CUSUM (C), respectively. An expression which summarizes this combination is given by

$$\alpha_{cs} = \alpha_s + \alpha_c - \alpha_s\alpha_c. \quad (4)$$

As such, two control charts with individual false alarm rates of 1% for example generate a combined chart with a false alarm rate of nearly 2%. Looking to maintain the combined rate at its original value of 1%, the control limits should be recalculated, resulting in values which are at least as distant from the center line as before and appropriately more [26] [27].

III. METHODOLOGY

The methodology used in this study has a quantitative approach involving simulation models. For the procedure of the combined chart, Monte Carlo simulations are used. As for the CUSUM charts, ARL values are calculated by Markov chains. ARL values for the Shewhart chart were also obtained by simulation.

Situations both in and out of statistical control were created, with the main characteristic under observation being large changes in the rate of defectives, and comparing the ARLs of combined charts, individual CUSUM charts and individual Shewhart charts. The individual CUSUM charts were designed to detect increases of 1.25 (25%) up to 3 times (200%) the proportion p_0 considered in statistical control, see table 1. The percentage change in p_0 will be called s. Statistical analysis and simulations were all performed using R [28] with the help of the surveillance package [29].

IV. RESULTS

Initially, the performance of the combined Shewhart-CUSUM chart and individual CUSUM charts designed to detect larger values of p_1 will be compared. The combined chart is designed to quickly detect a value of p_1 = 1.25 p_0, i.e. an increase of s = 25% in the value of the proportion under control, with k = 6.3, m = 24 and UCL = 15. The individual CUSUM charts have reference value k calculated to detect proportions $p_1 = dp_0$, i.e., p_1 = 1.25p_0, 1.5p_0, 2p_0 and 3p_0. The following values were also used: p_0 = 0.02, n = 50 and ARL_0 = 200. The parameters of the charts analyzed are shown in Table 1.

Table 2 has the ARL values relative to the combined charts and individual CUSUM charts mentioned above. Figure 1 has the comparative log (ARL) charts of the combined and individual CUSUM chart for p_1 = 1.25p_0. The use of the logarithm improves the visualization of the chart.

Figure 1 shows that the CUSUM chart and the combined chart have virtually the same performance for minor changes (approximately $p_1 \leq 1.5p_0$), and as the changes increase, the combined starts to outperform the CUSUM.

Figure 2 shows the ARL values of the combined chart and the CUSUM chart (CUSUM 1.5) with p_1 = 1.5p_0 (s = 50%).

Table 1: Parameters of the Combined Shewhart-CUSUM chart and individual CUSUM charts for ARL_0 = 200

Chart	k	h	UCL
Combined Shewhart-CUSUM	1.12	8.6	5.0
CUSUM 1.25 (s = 25%)	1.12	8.3	
CUSUM 1.5 (s = 50%)	1.23	6.4	
CUSUM 2.0 (s = 100%)	1.40	5.0	
CUSUM 3.0 (s = 200%)	1.83	3.2	

Table 2: ARL values relative to the combined charts and individual CUSUM charts for $p_0 = 0.02$, $n = 50$ and $ARL_0 = 200$. Bold numerals are the lowest values in each row. Italics means that CUSUM dominates combined charts.

d	Comb.	CUSUM1.25 k=1.12	CUSUM1.5 k=1.32	CUSUM2.0 k=1.40	CUSUM3.0 k=1.83
1.00	204.00	204.37	203.77	208.4	211.70
1.10	97.16	***94.44***	99.19	109.33	126.70
1.25	44.16	***43.86***	45.27	50.98	65.75
1.50	21.11	21.27	***20.36***	21.51	28.11
1.75	13.51	13.90	*12.67*	*12.48*	15.14
2.00	9.96	10.36	*9.18*	***8.62***	*9.60*
2.25	7.85	8.27	*7.23*	*6.58*	*6.82*
2.50	6.41	6.89	*5.98*	*5.34*	*5.23*
2.75	5.45	5.90	*5.11*	*4.51*	*4.24*
3.00	4.71	5.17	*4.47*	*3.92*	***3.57***
3.25	4.17	4.61	*3.98*	*3.48*	*3.10*
3.50	3.72	4.18	*3.59*	*3.13*	*2.75*
4.00	3.01	3.54	3.01	*2.63*	*2.28*
4.5	2.54	3.09	2.61	*2.28*	*1.97*
5.0	2.17	2.76	2.31	*2.03*	*1.76*
7.5	1.26	1.89	1.55	1.38	*1.22*

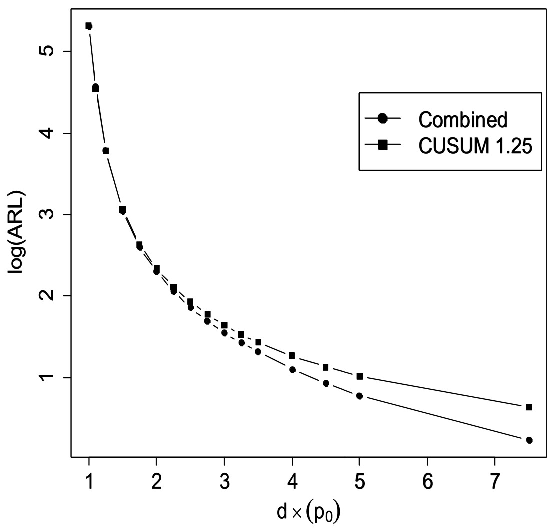

Figure 1: Comparison of the log (ARL) of the combined and CUSUM (1.25) charts for $p_0 = 0.02$, $n = 50$ and $ARL_0 = 200$

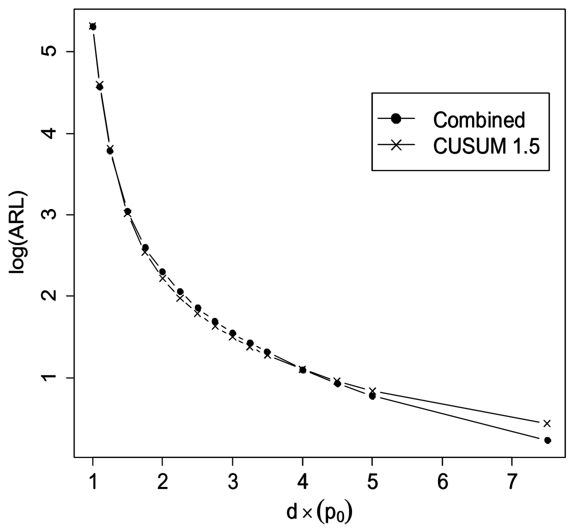

Figure 2: Comparison of the log (ARL) of the combined and CUSUM (1.5) charts for $p_0 = 0.02$, $n = 50$ and $ARL_0 = 200$

For moderate changes, near p_1 of the individual CUSUM, the CUSUM shows the better performance. The combined only outperforms the CUSUM for changes larger than $p_1 = 4p_0$ and as it nears $p_1 = 1.25p_0$ (Figure 2).

Figure 3 has the ARL values of the combined and CUSUM charts (CUSUM 2.0) for $p_1 = 2p_0$ (s = 100% in p_0). The combined chart is more effective only for small changes, $p_1 < 1.5\ p_0$ and very large changes, approximately $p_1 > 6\ p_0$.

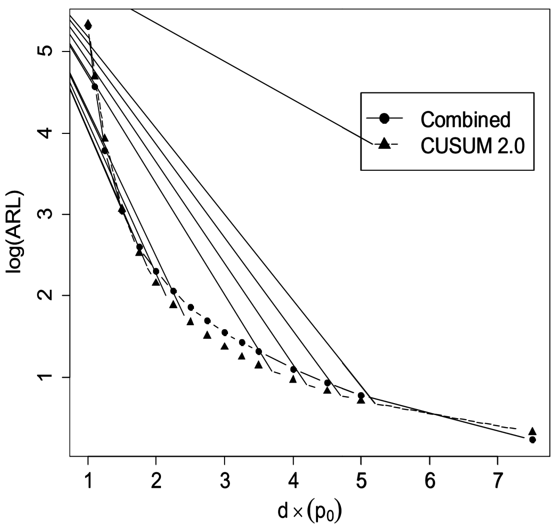

Figure 3: Comparison of the log (ARL) of the combined and CUSUM 2.0 charts for p_0 = 0.02, n = 50 and ARL_0 = 200

In Figure 4, the ARL values of the combined chart and CUSUM chart (CUSUM 3.0) can be compared for $p_1 = 3p_0$ (s = 200%). The individual CUSUM has better performance for shifts over $p_1 = 2p_0$. The combined is only more sensitive to minor shifts, $p_1 < 2p_0$ here.

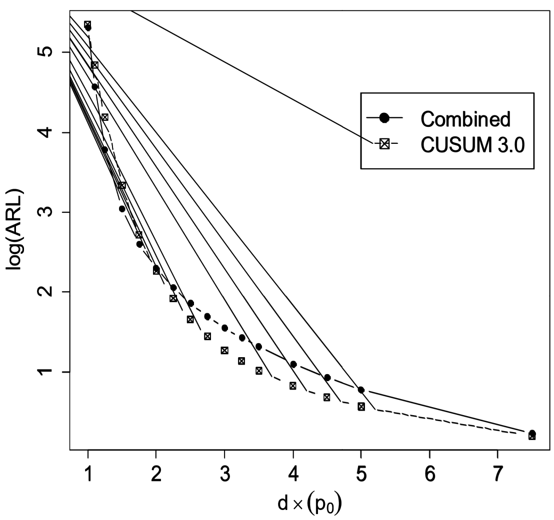

Figure 4: Comparison of the log (ARL) of the combined and CUSUM 3.0 charts for p_0 = 0.02, n = 50 and ARL_0 = 200

Figures 5, 6, 7 and 8 show the rARL analyses. The rARL a measurement suggested by [30] in order to facilitate comparing ARL values among diverse charts. With this measurement, the ARL of the combined chart will be compared with respect to the ARL of the CUSUM for a specific shift in the value of proportion p. Values of rARL less than one (1) imply that the combined chart is more efficient than the simple CUSUM in detecting a shift in a particular p.

$$rARL(p) = \frac{ARL_{combined}(p)}{ARL_{CUSUM}(p)} . \quad (4)$$

Through these comparative charts, the earlier findings are reinforced. The performance of the combined in detection of larger changes is lower when compared with the CUSUM procedures designed to detect those particular changes. Figure 5 shows that the combined chart has superior performance when compared to the CUSUM (1.25) chart for shifts greater than 50% (1,50p_0).

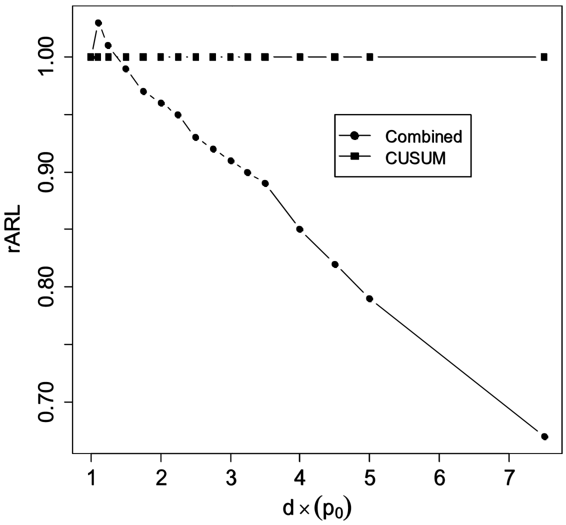

Figure 5 - Comparison of the rARL of the combined and individual CUSUM charts for p0 = 0.02, n = 50 and
ARL_0 = 200

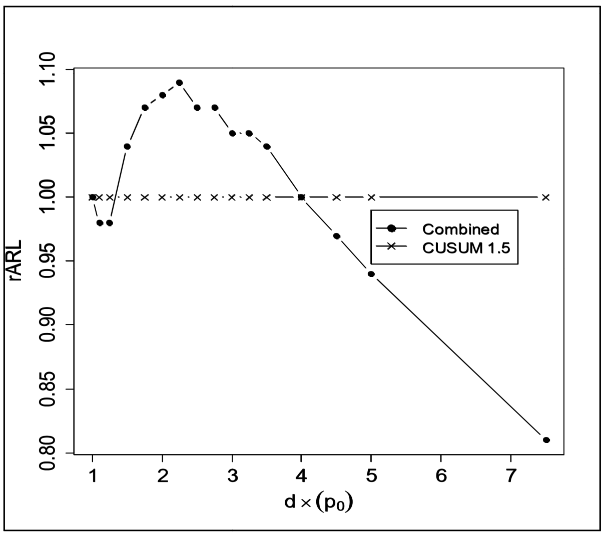

Figure 6 - Comparison of the rARL of the combined and and CUSUM 1.5 charts for p0 = 0.02, n = 50 and ARL_0 = 200

In Figure 6 it can be seen that the combined chart is more effective than the CUSUM (1.5) for very small changes (less than $1.25p_0$), and great shifts, more than $4p_0$.

Figure 7 shows that the combined chart performs better than the CUSUM (2.0) for shifts, less than about $1.5p_0$ and more than $6p_0$. In Figure 8 is visualized that the combined chart is more effective than the CUSUM (3.0) only for changes below about $2p_0$ and very large shifts, not shown in this figure.

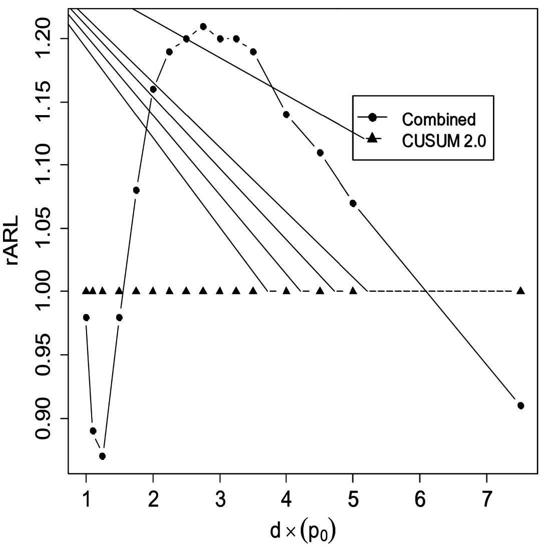

Figure 7 - Comparison of the rARL of the combined and and CUSUM 2.0 charts for p0 = 0.02, n = 50 and ARL_0 = 200

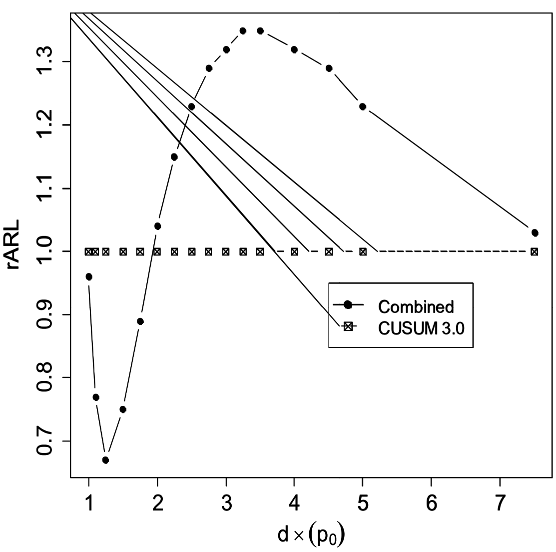

Figure 8 - Comparison of the rARL of the combined and and individual CUSUM 3.0 charts for p0 = 0.02, n = 50 and ARL_0 = 200

In the next example, the combined chart was designed to quickly detect a value of $p_1 = 1.1p_0$. The individual CUSUM charts have reference value k calculated to detect equal proportions p_1 equal to $1.1p_0$, $1.5p_0$ and $2p_0$. The following values were also used: $p_0 = 0.1$, $p_1 = 0.11$, $n = 100$ and $ARL_0 = 50$. The parameters of the charts are shown in Table 3.

Table 3: Chart parameters $ARL_0 = 50$

Tipo	*k*	*h*	UCL
Combinado	10.49	12.6	19
CUSUM 1.1(s = 10%)	10.49	12.5	
CUSUM 1.5 (s = 50%)	12.35	5.0	
CUSUM 2.0 (s =100%)	14.52	2.4	
Shewhart			16.0

Table 4 shows the ARL values obtained. In Figure 9, there are the comparative graphs of the log (ARL) for the combined and individual CUSUM procedures with the above specifications in Table 3. In this case, the value of the UCL of the combined is 19 and the value of h is 12.6. It can be seen that the combined chart has performance similar to the CUSUM (10%) for small and moderate changes.

It can be seen that the combined chart has performance similar to the CUSUM (s = 10%) for small and moderate shifts. The combined chart is more effective for larger shifts with values of s greater than 50% approximately (Table 4 and Figure 9). And in this case, for changes on the order of $p_1 = 1.4p_0$ (Table 4), the combined chart outperforms both the individual CUSUM chart and the Shewhart chart. In fact, it can be observed that for each situation there seems to exist a small range of values where the combined chart's performance is superior to that of the individual schemes.

This range covers values furthest from the magnitude of the desired change of the CUSUM, and those values are not very large. This situation can be further investigated. The CUSUM procedures designed to detect larger shifts (s = 50 and 100%) are more effective than the combined chart, specifically for these changes (Table 4 and Figures 10 and 11). In Figure 12, besides comparing the combined chart with the CUSUM 2.0 (s = 100% in p_0), these are compared with a Shewhart procedure, in terms of ARL. It appears that the performance of the Shewhart chart is similar to the CUSUM designed to detect a 100% increase on the process mean.

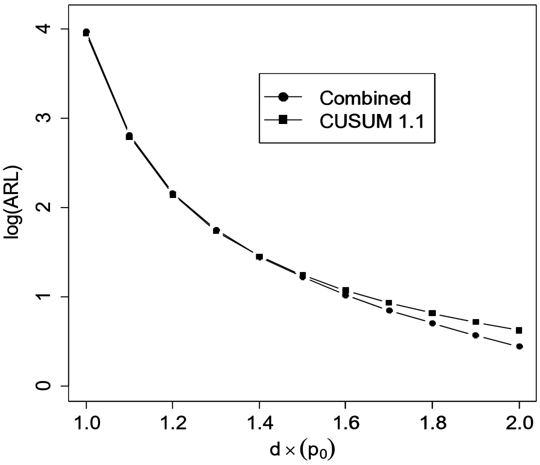

Figure 9: Comparison of the log (ARL) of the combined and CUSUM 1.1 charts for $p_0 = 0.1$, n = 100 and $ARL_0 = 50$

Table 4: ARL values relative to the combined charts and individual CUSUM charts for $p_0 = 0.1$, n = 100 and $ARL_0 = 50$ Bold numerals are the lowest values in each row. Italics means that CUSUM or Shewhart dominates combined charts.

d	comb	CUSUM1.1	CUSUM1.5	CUSUM2.0	Shewhart
1.0	53.29	52.08	50.27	47.50	48.90
1.1	16.58	***16.31***	19.01	21.21	22.06
1.2	8.69	***8.52***	9.12	10.93	11.47
1.3	5.77	*5.68*	***5.33***	6.37	6.66
1.4	4.26	4.28	***3.61***	*4.12*	4.29
1.5	3.39	3.46	***2.70***	*2.92*	*3.05*
1.6	2.78	2.92	***2.16***	*2.22*	*2.29*
1.7	2.34	2.54	*1.82*	***1.80***	*1.86*
1.8	2.02	2.26	*1.59*	***1.53***	*1.56*
1.9	1.77	2.05	*1.42*	***1.35***	*1.37*
2.0	1.56	1.87	*1.3*	***1.23***	*1.24*

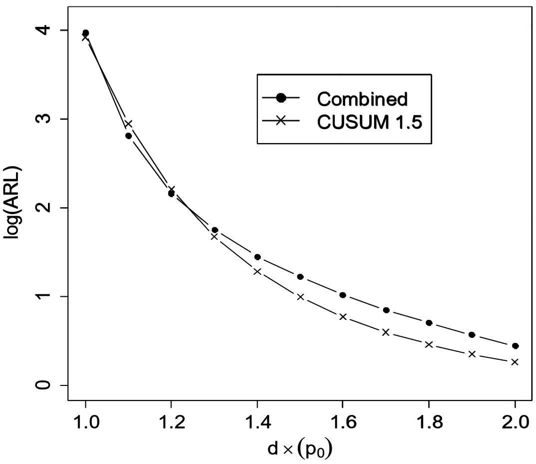

Figure 10: Comparison of the log (ARL) of the combined and CUSUM 1.5 charts for $p_0 = 0.1$, n = 100 and $ARL_0 = 50$

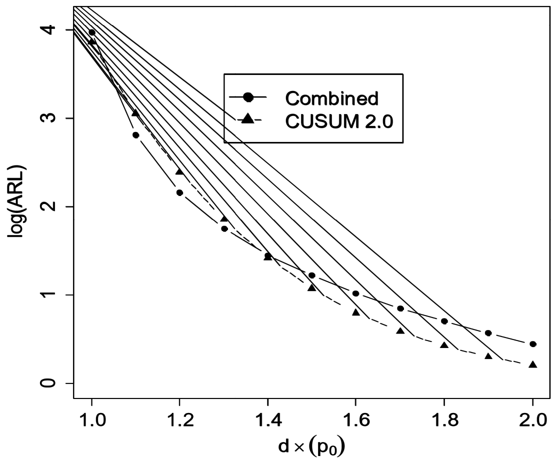

Figure 11: Comparison of the log (ARL) of the combined, and CUSUM 2.0 charts for $p_0 = 0.1$, n = 100 and $ARL_0 = 50$

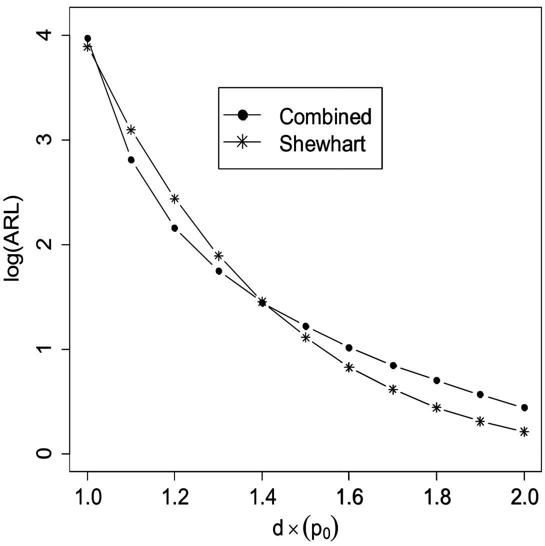

Figure 12: Comparison of the log (ARL) of the combined, and Shewhart charts for $p_0 = 0.1$, $n = 100$ and $ARL_0 = 50$

So, the combined chart outperforms the individual CUSUM for changes of greater magnitude, when compared with the individual CUSUM chart with the same p_1 design proportion. Regarding the other CUSUM charts, which are designed to detect larger changes, their performance is lower, in the range of the values analyzed.

V. CONCLUSIONS

Traditional Shewhart control charts are considered effective in the detection of significant changes in the mean of the process, whereas cumulative sum (CUSUM) control charts are suitable for detecting small and moderate changes. However none of the charts mentioned will perform well in all situations. To solve this problem, we recommend to combine multiple charts to cover changes of various magnitudes. The combined Shewhart-CUSUM chart is intended to increase the sensitivity of the CUSUM procedure for larger changes.

In this paper, the combined chart was compared in terms of ARL, to other CUSUM procedures designed to detect larger changes, based on the results of simulations. Two analyses were performed: one on the combined chart compared to individual CUSUM procedures and another one on the combined chart compared to the Shewhart procedure. The combined chart increases sensitivity of a CUSUM to changes larger than those for which the chart is designed. Thus, if the process requires detect sooner a particular shift, small or moderate, a combined chart is a suitable option, since it allows to detect the shifts set out in the planning, and offers the possibility of detecting major changes. Thus, if the process requires rapid identification of change, small or moderate, a combined chart is a suitable option, since it allows to detect the changes set out in the planning and offers the possibility of detecting major changes.If the process only requires a quick identification of a specific large shift, other procedures are recommended, such as the individual CUSUM or a Shewhart chart.

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